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Coupled Soft Fixed Point Theorems in Partially Ordered Soft Metric Space

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ABSTRACT: There is several generalization of Banach contraction principle. Recently Bhaskaran and Lakshmikantham generalized this result and prove coupled fixed point theorems in ordered metric space. In this present work, we proof some coupled soft fixed point theorems in ordered soft metric space.

Key words: Ordered Metric Space, Fixed point, Coupled soft Fixed point, mixed monotone property.

I. INTRODUCTION

In 1999 Molodtsov [24] introduced the concept of soft sets which is one of the branches of mathematics, which aims to describe phenomena and concepts of an ambiguous, vague, undefined and imprecise meaning. Shabir and Naz [34] introduced the notion of soft topology on a soft set and proved basic properties concerning soft topological spaces. Soft mapping space and soft uniform space were studied in [29, 30]. Xie introduced the notion of soft point and described the relationship between soft points and soft sets in [36]. Since soft sets cannot be handled like ordinary sets as they are defined by mappings. So defining a soft point rely both on the structure of the soft set and the mapping, which in turn give rise to different opportunities for defining a soft point. Das and Samanta introduced the notion of soft element by using a function, by using soft element they introduced soft real number [13]. Also Xie [36] introduced the notion of soft point in a different approach.

II. PRELIMINARIES

The purpose of this paper is to make contribution for investigating on soft metric spaces and thus we focus on partially ordered sets in soft metric spaces and explore the differences and similarities between the point set topology and soft topology. In this paper, we will find new coupled fixed point theorems for mapping having the mixed monotone property in partially ordered soft metric space.

Definition 2.1. [24] Let Dbe a set of parameters and Ebe an initial universe. Let $\mathrm{P(E)}$ denote the power set of E. A pair $(\mathrm{F,D})$ is called a soft set over E, where F is a mapping given by $\mathrm{F: D \rightarrow P(E)}$.

Definition 2.2. [13] Let $(\mathrm{F,D})$ and $(\mathrm{G,D})$ be two soft sets over a common initial universe E.

(a)$(\mathrm{F,D})$ is said to be null soft set (denoted by ϕ), if $\forall\alpha \in \mathrm{D}, \mathrm{F}(\alpha) = \phi$. And $(\mathrm{F,D})$ is said to be an absolute soft set (denoted by $\widetilde{\mathrm{E}}$), if $\forall\alpha \in \mathrm{D}, \mathrm{F}(\alpha) = \mathrm{E}$.

(b) The union of $(\mathrm{F,D})$ and $(\mathrm{G,D})$ overE is $(\mathrm{H,D})$ defined as $\mathrm{H}(\alpha) = \mathrm{F}(\alpha) \cup \mathrm{G}(\alpha), \forall\, \alpha \in \mathrm{D}$. We write $(\mathrm{F,D})\ \widetilde{\cup}\ (\mathrm{G,D}) = (\mathrm{H,D})$.

Definition 2.3 [14] Let Dbe a non-empty parameter set and Zbe a non-empty set. Then a function $\mathrm{h: D \rightarrow Z}$ is said to be a soft element of Z. A soft element h of Z is said to belongs to a soft set $(\mathrm{F,D})$ of Z which is denoted by $\mathrm{h}\ \widetilde{\in}\ (\mathrm{F,D})$ if $\mathrm{h}(\alpha) \in \mathrm{F}(\alpha), \forall\, \alpha \in \mathrm{D}$. Thus for a soft set $(\mathrm{F,D})$ of Z with respect to the index set D, we have $\mathrm{F}(\alpha) = \{\mathrm{h}(\alpha){:}\, \mathrm{h}\ \widetilde{\in}\ (\mathrm{F,D}), \alpha \in \mathrm{D}$. In that case, his also said to be a soft element of the soft set $(\mathrm{F,D})$. Thus every singlet on soft set (a soft set $(\mathrm{F,D})$ of E for which $\mathrm{F}(\alpha)$ is a singlet on set, $\forall\, \alpha \in \mathrm{D}$) can be identified with a soft element by simply identifying the singleton set with the element that it contains $\forall\, \alpha \in \mathrm{D}$.

Definition 2.4. [13,14] Let R be the set of real numbers and Dbe a set of parameters and $\mathrm{B(R)}$ be the collection of non-empty bounded subsets of R. Then a mapping $\mathrm{F: D \rightarrow B(R)}$ is called a soft real set, denoted by $(\mathrm{F,D})$. If specifically $(\mathrm{F,D})$ is a singleton soft set, then after identifying $(\mathrm{F,D})$with the corresponding soft element, it will be called a soft real number. The set of all soft real numbers is denoted by $\mathrm{R(D)}$ and the set of non-negative soft real numbers by $\mathrm{R(D)}^*$.

Let $\tilde{r}$ and $\tilde{s}$ be two soft real numbers. Then the following statements hold:

- $\tilde{r} \widetilde{\leq} \tilde{s}$, if $\tilde{r}(\alpha) \leq \tilde{s}(\alpha), \forall\, \alpha \in \mathrm{D}$,
- $\tilde{r} \widetilde{>} \tilde{s}$, if $\tilde{r}(\alpha) > \tilde{s}(\alpha), \forall\, \alpha \in \mathrm{D}$,
- $\tilde{r} \widetilde{\geq} \tilde{s}$, if $\tilde{r}(\alpha) \geq \tilde{s}(\alpha), \forall\, \alpha \in \mathrm{D}$,
- $\tilde{r} \widetilde{<} \tilde{s}$, if $\tilde{r}(\alpha) < \tilde{s}(\alpha), \forall\, \alpha \in \mathrm{D}$.

Proposition 2.5. [14] (a) For any soft sets $(F,D),(G,D) \in S(\widetilde{E})$, we have $(F,D) \tilde{\subseteq} (G,D)$ if and only if every soft element of (F,D) is also a soft elements of (G,D).

(b) Any collection of soft elements of a soft set can generate a soft subset of that soft set. The soft set constructed from a collection Bof soft elements is denoted by $SS(B)$.

(c) For any soft set $(F,D) \in S(\widetilde{E}), SS(SE(F,D)) = (F,D)$; whereas for a collection B of soft elements, $SE(SS(B)) \supset B$, but, in general, $SE(SS(B)) \neq B$.

Definition 2.6. [15, 16]

(a) A sequence $\{\widetilde{x_n}\}$ of soft elements in a soft normed linear space $(\widetilde{X}, \|.\|, A)$is said to be convergent and converges to a soft element $\tilde{x}$ if $\| \widetilde{x_n} - \tilde{z} \| \to \bar{0}$ as $n \to \infty$. This means for every $\tilde{\epsilon} \tilde{>} \bar{0}$, chosen arbitrarily, $\exists$ a natural number $N = N(\tilde{\epsilon})$ such that $\bar{0} \tilde{\leq} \| \widetilde{x_n} - \tilde{x} \| \tilde{<} \tilde{\epsilon}$ whenever $n > N$i.e. $n > N \Rightarrow \widetilde{x_n} \in B(\tilde{x}, \tilde{\epsilon})$, (where $B(\tilde{x}, \tilde{\epsilon})$ is an open ball with centre $\tilde{x}$ and radius $\tilde{\epsilon}$).

(b) A sequence $\{\widetilde{x_n}\}$ of soft elements in a soft normed linear space $(\widetilde{Z}, \|.\|, A)$ is said to be a Cauchy sequence in $\widetilde{X}$ if corresponding to every $\tilde{\epsilon} \tilde{>} \bar{0} \exists$ a natural number $N = N(\tilde{\epsilon})$ such that $\| \widetilde{x_n} - \widetilde{x_m} \| \tilde{\leq} \tilde{\epsilon}, \forall\, m, n > N$ i.e. $\| \widetilde{x_n} - \widetilde{x_m} \| \to \bar{0}$ as $n, m \to \infty$.

(c) Let $(\widetilde{X}, \|.\|, D)$ be a soft normed linear space. Then $\widetilde{X}$ is said to be complete if every Cauchy sequence of soft elements in $\widetilde{X}$ converges to a soft element of $\widetilde{X}$. Every complete soft normed linear space is called a soft Banach space.

Definition 2.7. [14] Let $\widetilde{X}$ be a non-empty set and Dbe non-empty a parameter set. A mapping $d: SV(\widetilde{X}) \times SV(\widetilde{X}) \to R(D)^*$ is said to be a soft metric on the soft set $\widetilde{X}$ if d satisfies the following conditions:

- $d(\tilde{x}, \tilde{y}) \tilde{\geq} \bar{0}, \forall\, \tilde{x}, \tilde{y} \in \widetilde{X}$.
- $d(\tilde{x}, \tilde{y}) = \bar{0}$, if and only if $\tilde{x} = \tilde{y}$.
- $d(\tilde{x}, \tilde{y}) = d(\tilde{y}, \tilde{x}), \forall\, \tilde{x}, \tilde{y} \in \widetilde{X}$.
- $d(\tilde{x}, \tilde{y}) \tilde{\leq} d(\tilde{x}, \tilde{z}) + d(\tilde{z}, \tilde{y}), \forall\, \tilde{x}, \tilde{y}, \tilde{z} \in \widetilde{X}$.

The soft $\widetilde{X}$ with a soft metric don $\widetilde{X}$ is said to be a soft metric space and denoted by $(\widetilde{X}, d, D)$ or $(\widetilde{X}, d)$.

Proposition 2.8. [16] Let $(\widetilde{X}, \|.\|, D)$ be soft normed linear space. Let us defined: $\widetilde{X} \times \widetilde{X} \to R(D)^*$ by $d(\tilde{x}, \tilde{y}) = k\tilde{x} - \tilde{y}k, \forall\, \tilde{x}, \tilde{y} \in \widetilde{X}$. Then dis a soft metric on $\widetilde{X}$.

Definition 2.8. A partially ordered set is a soft set P with binary relation $\leq$, denoted by $(\widetilde{X}, \leq)$ such that $\forall a, b, c \in P$

- $a \leq a$, (reflexivity),
- $a \leq b$ and $b \leq c \Rightarrow a \leq c$, (transitivity),
- $a \leq$ band $b \leq a \Rightarrow a = b$, (anti-symmetry).

Definition 2.9. A sequence $\{\widetilde{x_n}\}$ in a soft metric space $(\widetilde{X}, d)$ is said to be convergent to a point $\tilde{x} \in \widetilde{X}$denoted by$\lim_{n\to\infty} \widetilde{x_n} = \tilde{x}$, if $\lim_{n\to\infty} d(\widetilde{x_n}, \tilde{x}) = 0$. and is said to be soft Cauchy sequence if $\lim_{n\to\infty} d(\widetilde{x_n}, \widetilde{x_n}) = 0, \forall\, n, m > t$.A soft metric space $(\widetilde{X}, d)$ is said to be complete if every Cauchy sequence in $\widetilde{X}$ is convergent.

Definition 2.10. Let $(\widetilde{X}, <)$be a partially ordered soft set, let $F: \widetilde{X} \times \widetilde{X} \to \widetilde{X}$. The mapping F is said to have the mixed monotone property if $F(\tilde{x}, \tilde{y})$ is non-decreasing in $\tilde{x}$ and is said to be monotonenon-increasing in $\tilde{y}$, that is, for any$\tilde{x}, \tilde{y} \in \widetilde{X}$,

$\widetilde{x_1}, \widetilde{x_2} \in \widetilde{X}, \widetilde{x_1} \leq \widetilde{x_2} \Rightarrow F(\widetilde{x_1}, \tilde{y}) \leq F(\tilde{y}, \widetilde{x_1})$ and $\widetilde{y_1}, \widetilde{y_2} \in \widetilde{X}, \widetilde{y_1} \leq \widetilde{y_2} \Rightarrow F(\tilde{x}, \widetilde{y_1}) \geq F(\tilde{x}, \widetilde{y_2})$

Definition 2.11. An element $(\tilde{x}, \tilde{y}) \in \widetilde{X} \times \widetilde{X}$ is called a coupled soft fixed point of the mapping $F: \widetilde{X} \times \widetilde{X} \to \widetilde{X}$ if $\tilde{x} = F(\tilde{x}, \tilde{y})$ and $\tilde{y} = F(\tilde{y}, \tilde{x})$.

Theorem 2.7 Let $(\widetilde{X}, \leq)$ be a partially ordered soft set and suppose there exists a metric d on $\widetilde{X}$ such that $(\widetilde{X}, d)$is a complete soft metric space. Let$F: \widetilde{X} \times \widetilde{X} \to \widetilde{X}$ be a continuous mapping having the mixedmonotone property on $\widetilde{X}$. Assume that there exists a$\alpha \in [0,1)$ with

For all if there exist two elements with

$$d(F(\tilde{x}, \tilde{y}), F(\tilde{u}, \tilde{v})) \leq \frac{\alpha}{2}[(F(\tilde{x}, \tilde{y}) + d(\tilde{u}, \tilde{v})]$$

$\forall \tilde{x} \geq \tilde{u}$ and $\tilde{y} \leq \tilde{v}$ if there exist two elements $\widetilde{x_0}, \widetilde{y_0} \in \widetilde{X}$ with $\widetilde{x_0} \leq F(\widetilde{x_0}, \widetilde{y_0})$ and $\widetilde{y_0} \geq F(\widetilde{y_0}, \widetilde{x_0})$ then, there exist$\tilde{x}, \tilde{y} \in \widetilde{X}$ such that $\tilde{x} = F(\tilde{x}, \tilde{y})$ and $\tilde{y} = F(\tilde{y}, \tilde{x})$.

III. MAIN RESULTS

Theorem 3.1. Let $(\widetilde{X}, \leq)$ be a soft partially ordered set and suppose there exists a soft metric d on $\widetilde{X}$ such that $(\widetilde{X}, d)$ is a complete soft metric space. Let $F: \widetilde{X} \times \widetilde{X} \to \widetilde{X}$ be a continuous mapping having the mixedmonotone property on $\widetilde{X}$. Assume that there exists a$\alpha \in [0,1)$ with

$$d(F(\tilde{x}, \tilde{y}), F(\tilde{u}, \tilde{v})) \leq \alpha \max\{\frac{d(\tilde{x}, F(\tilde{x}, \tilde{y})), d(\tilde{u}, F(\tilde{u}, \tilde{v}))}{d(\tilde{x}, \tilde{u})}, \frac{d(\tilde{u}, F(\tilde{x}, \tilde{y})), d(\tilde{x}, F(\tilde{u}, \tilde{v}))}{d(\tilde{x}, \tilde{u})}, d(\tilde{x}, \tilde{u})\}$$

$\forall \tilde{x} \geq \tilde{u}$ and $\tilde{y} \leq \tilde{v}$ if there exist two elements $\widetilde{x_0}, \widetilde{y_0} \in \widetilde{X}$ with $\widetilde{x_0} \leq F(\widetilde{x_0}, \widetilde{y_0})$and $\widetilde{y_0} \geq F(\widetilde{y_0}, \widetilde{x_0})$ then, there exist $\tilde{x}, \tilde{y} \in \widetilde{X}$ such that $\tilde{x} = F(\tilde{x}, \tilde{y})$ and $\tilde{y} = F(\tilde{y}, \tilde{x})$.

Proof. Let $\widetilde{x_0}, \widetilde{y_0} \in \widetilde{X}$ with

$\widetilde{x_0} \leq F(\widetilde{x_0}, \widetilde{y_0})$ & $\widetilde{y_0} \geq F(\widetilde{y_0}, \widetilde{x_0})$(3.1.2)

Define the sequence $\{\widetilde{x_n}\}$ and $\{\widetilde{y_n}\}$ in $\widetilde{X}$ such that,

$\widetilde{x_{n+1}} = F(\widetilde{x_n}, \widetilde{y_n})$ & $\widetilde{y_{n+1}} = F(\widetilde{y_n}, \widetilde{x_n})$ (3.1.3) $\forall\, n = 0,1,2, \dots\dots\dots$

We claim that $\{\widetilde{x_n}\}$ is monotone non decreasing and $\{\widetilde{y_n}\}$monotone non increasing i.e., $\widetilde{x_n} \leq \widetilde{x_{n+1}}$and $\widetilde{y_n} \geq \widetilde{y_{n+1}}$(3.1.4) $\forall\, n = 0,1,2, \dots\dots\dots$

From (3.1.2) and (3.1.3) we have
$\widetilde{x_0} \leq F(\widetilde{x_0}, \widetilde{y_0}), \widetilde{y_0} \geq F(\widetilde{y_0}, \widetilde{x_0})$ and $\widetilde{x_1} = F(\widetilde{x_0}, \widetilde{y_0})$, $\widetilde{y_1} = F(\widetilde{y_0}, \widetilde{x_0})$
Thus $\widetilde{x_0} \leq \widetilde{x_1}, \widetilde{y_0} \geq \widetilde{y_1}$ i.e., equation (3.1.4) is true for some $n = 0$
Now suppose that equation (3.1.4) holds for some n.

i.e, $\widetilde{x_n} \leq \widetilde{x_{n+1}}$ and $\widetilde{y_n} \geq \widetilde{y_{n+1}}$

We shall prove that the equation (3.1.4) is true for $n+1$
Now $\widetilde{x_n} \leq \widetilde{x_{n+1}}$ and $\widetilde{y_n} \geq \widetilde{y_{n+1}}$ then by mixed monotone property of F, we have

$$\widetilde{x_{n+2}} = F(\widetilde{x_{n+1}}, \widetilde{y_{n+1}}) \geq F(\widetilde{y_n}, \widetilde{x_{n+1}}) \geq F(\widetilde{x_n}, \widetilde{y_n}) = \widetilde{x_{n+1}}$$

and $\widetilde{y_{n+2}} = F(\widetilde{y_{n+1}}, \widetilde{x_{n+1}}) \leq F(\widetilde{y_n}, \widetilde{x_{n+1}}) \leq F(\widetilde{y_n}, \widetilde{x_n}) = \widetilde{y_{n+1}}$
Thus by the mathematical induction principle equation (3.1.4) holds for all n in N.
So $\widetilde{x_0} \leq \widetilde{x_1} \leq \widetilde{x_2} \leq \ldots\ldots\ldots \leq \widetilde{x_n} \leq \widetilde{x_{n+1}} \leq \ldots\ldots\ldots$
and $\widetilde{y_0} \geq \widetilde{y_1} \geq \widetilde{y_2} \geq \ldots\ldots\ldots \geq \widetilde{y_n} \geq \widetilde{y_{n+1}} \geq \ldots\ldots\ldots$
since $\widetilde{x_{n-1}} \leq \widetilde{x_n}$ and $\widetilde{y_{n-1}} \geq \widetilde{y_n}$, from (3.1.1) we have,

$$d\left(F(\widetilde{x_n}, \widetilde{y_n}), F(\widetilde{x_{n-1}}, \widetilde{y_{n-1}})\right) \leq \alpha \max\left\{\frac{d\left(\widetilde{x_n}, F(\widetilde{x_n}, \widetilde{y_n})\right), d\left(\widetilde{x_{n-1}}, F(\widetilde{x_{n-1}}, \widetilde{y_{n-1}})\right)}{d(\widetilde{x_n}, \widetilde{x_{n-1}})}, \frac{d\left(\widetilde{x_{n-1}}, F(\widetilde{x_n}, \widetilde{y_n})\right), d\left(\widetilde{x_n}, F(\widetilde{x_{n-1}}, \widetilde{y_{n-1}})\right)}{d(\widetilde{x_n}, \widetilde{x_{n-1}})}, d(\widetilde{x_n}, \widetilde{x_{n-1}})\right\}$$

$$d(\widetilde{x_{n+1}}, \widetilde{x_n}) \leq \alpha \max\{d(\widetilde{x_n}, \widetilde{x_{n+1}}), 0, d(\widetilde{x_n}, \widetilde{x_{n-1}})\}$$

If we take, max is equal to $d(\widetilde{x_n}, \widetilde{x_{n+1}})$,
$d(\widetilde{x_n}, \widetilde{x_{n+1}}) \leq \alpha d(\widetilde{x_n}, \widetilde{x_{n-1}})$, which contradiction of the hypothesis,
This implies, $d(\widetilde{x_n}, \widetilde{x_{n+1}}) \leq \alpha d(\widetilde{x_n}, \widetilde{x_{n-1}})$ (3.1.5)
Similarly, from $\widetilde{y_{n-1}} \geq \widetilde{y_n}$, $\widetilde{x_{n-1}} \leq \widetilde{x_n}$ and from (3.1.1) we have
$d(\widetilde{y_n}, \widetilde{y_{n+1}}) \leq \alpha d(\widetilde{y_n}, \widetilde{y_{n-1}})$ (3.1.6)
By adding (3.1.5) and (3.1.6) we get,

$$d(\widetilde{x_n}, \widetilde{x_{n+1}}) + d(\widetilde{y_n}, \widetilde{y_{n+1}}) \leq \alpha d(\widetilde{x_n}, \widetilde{x_{n-1}}) + \alpha d(\widetilde{y_n}, \widetilde{y_{n-1}})$$
$$\Longrightarrow d(\widetilde{x_n}, \widetilde{x_{n+1}}) + d(\widetilde{y_n}, \widetilde{y_{n+1}}) \leq \alpha[d(\widetilde{x_n}, \widetilde{x_{n-1}}) + d(\widetilde{y_n}, \widetilde{y_{n-1}})]$$

let us denoted $(\widetilde{x_n}, \widetilde{x_{n+1}}) + d(\widetilde{y_n}, \widetilde{y_{n+1}})$ by d_n, then $d_n \leq \alpha d_{n-1}$
Similarly it can be proved that $d_{n-1} \leq \alpha d_{n-2}$
Therefore, $d_n \leq \alpha^2 d_{n-2}$
By repeating we get, $d_n \leq \alpha d_{n-1} \leq \alpha^2 d_{n-2} \leq \ldots\ldots\ldots \leq \alpha^n d_0$
This implies that, $\lim_{n\to\infty} d_n = 0$
Thus $\lim_{n\to\infty} d(\widetilde{x_n}, \widetilde{x_{n+1}}) = \lim_{n\to\infty} d(\widetilde{y_n}, \widetilde{y_{n+1}}) = 0$
For each $m >$ □ we have

$$d(\widetilde{x_n}, \widetilde{x_m}) \leq d(\widetilde{x_n}, \widetilde{x_{n+1}}) - d(\widetilde{x_{n+1}}, \widetilde{x_{n+2}}) + \ldots\ldots\ldots + d(\widetilde{x_{m-1}}, \widetilde{x_m})$$

and

$$d(\widetilde{y_n}, \widetilde{y_m}) \leq d(\widetilde{y_n}, \widetilde{y_{n+1}}) + d(\widetilde{y_{n+1}}, \widetilde{y_{n+2}}) - \ldots\ldots\ldots + d(\widetilde{y_{m-1}}, \widetilde{y_m})$$

By adding these, we get

$$d(\widetilde{x_n}, \widetilde{x_m}) + d(\widetilde{y_n}, \widetilde{y_m}) \leq \frac{\alpha^n}{1-\alpha} d_0$$

This implies that,

$$\lim_{n,m\to\infty}[d(\widetilde{x_n}, \widetilde{x_m}) + d(\widetilde{y_n}, \widetilde{y_m})] = 0$$

Therefore, $\{\widetilde{x_n}\}$ and $\{\widetilde{y_n}\}$ are Cauchy sequence in $\widetilde{X}$. Since $\widetilde{X}$ is a complete metric space, there exist $\tilde{x}, \tilde{y} \in \widetilde{X}$, such that $\lim_{n\to\infty} \widetilde{x_n} = \tilde{x}$ and $\lim_{n\to\infty} \widetilde{y_n} = \tilde{y}$.
Thus by taking lim. $n \to \infty$ as in (3.1.3) we get,

$$\tilde{x} = \lim_{n\to\infty} \widetilde{x_n} = \lim_{n\to\infty} F(\widetilde{x_{n-1}}, \widetilde{y_{n-1}}) = F\lim_{n\to\infty}(\widetilde{x_{n-1}}, \widetilde{y_{n-1}}) = F(\tilde{x}, \tilde{y})$$

And

$$\tilde{y} = \lim_{n\to\infty} \widetilde{y_n} = \lim_{n\to\infty} F(\widetilde{y_{n-1}}, \widetilde{x_{n-1}}) = F\lim_{n\to\infty}(\widetilde{y_{n-1}}, \widetilde{x_{n-1}}) = F(\tilde{y}, \tilde{x})$$

Therefore $\tilde{x} = F(\tilde{x}, \tilde{y})$ and $\tilde{y} = F(\tilde{y}, \tilde{x})$
Thus F has a coupled fixed point in $\widetilde{X}$.

Theorem 3.2. Let $(\widetilde{X}, \leq)$ be a soft partially ordered set and suppose there exists a soft metric d on $\widetilde{X}$ such that $(\widetilde{X}, d)$ is a complete soft metric space. Let $F: \widetilde{X} \times \widetilde{X} \to \widetilde{X}$ be a continuous soft mapping having the mixed monotone property on $\widetilde{X}$. Assume that there exists a $\alpha \in [0,1)$ with
$d(F(\tilde{x}, \tilde{y}), F(\tilde{u}, \tilde{v})) \leq$
$\alpha \max\{d(\tilde{u}, F(\tilde{x}, \tilde{y})), d(\tilde{x}, F(\tilde{u}, \tilde{v}))\}$ (3.2.1)
$\forall\, \tilde{x} \geq \tilde{u}$ and $\tilde{y} \leq \tilde{v}$, if there exist two elements $\widetilde{x_0}, \widetilde{y_0} \in \widetilde{X}$ with $\widetilde{x_0} \leq F(\widetilde{x_0}, \widetilde{y_0})$ and $\widetilde{y_0} \geq F(\widetilde{y_0}, \widetilde{x_0})$ then, there exist $\tilde{x}, \tilde{y} \in \widetilde{X}$ such that $\tilde{x} = F(\tilde{x}, \tilde{y})$ and $\tilde{y} = F(\tilde{y}, \tilde{x})$.

Proof. Can be proved easily as above.

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